SIMPLE CLASSIFICATION TECHNIQUE FOR 3D HIGHLY HETEROGENEOUS DOMAINS AND ITS APPLICATION IN DEFIBRILLATION MODELING

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ABSTRACT
An efficient method is developed and implemented for classification of three dimensional objects with different material properties and highly irregular boundaries. The ray "tube" tracing technique proposed here was developed as a part of a software for automatic generation of unstructured three dimensional finite element grids. It operates on volumes tessellated in tetrahedral elements and region boundaries represented by surface triangles. This method is object independent and appropriate for convex and concave regions, with many depth levels. It is simple and gives a significant speedup (up to 15 times) compared to the traditional ray tracing by confining the search and the computationally expensive procedures to about 7% of all surface triangles for a given region. The "tube" construction, is based on the distance between the centroid of the surface primitives and the ray.

Complex structures with different conductivity properties are typical in large-scale finite element modeling in electrocardiology (including the torso, skeletal muscle, lungs, epicardium, heart with major blood vessels, different electrode configurations, etc.). The classification algorithm was successfully applied to a variety of problems calculating defibrillation induced potential fields using physiologically realistic geometries.

INTRODUCTION
The inherent complexity of the biological systems requires efficient modeling tools to study them. The improvement of defibrillation therapy in the last decade is partially due to the development of physiologically realistic models [1]. In these large-scale simulations finite element analysis is preferably performed on locally refined three-dimensional meshes [2]. The unstructured grids offer a general approach for tessellation of an arbitrary domain; they are not constrained to a specific curvilinear coordinate system.

In order to assign the proper conductivity tensor to each volume element in these highly heterogeneous conglomerates, efficient classification techniques are required, since they may be the time constraining part of the mesh generation process. Most often a ray tracing or solid angle based algorithm is chosen [2]. The general ray tracing method [3] does not involve trigonometric functions, thus it is preferred for speed.

The research on efficient ray trace algorithms has focussed on minimizing the cost of the ray-object intersection calculations. Different acceleration techniques have been suggested, primarily based on partitioning methods: hierarchical bounding [4, 5], where the region is partitioned into a tree of enclosing volumes; and space subdivision [6], which divides the objects into uniformly sized spatial cells. The efficiency of these approaches has been recognized but their implementation is not always simple, especially for non-convex objects with irregular boundaries. In addition, the correct termination point for the subdivision might be hard to determine [7].

The goal of this study was to develop a fast and simple region classification algorithm, that is independent of the structure and shape of the domain and is appropriate for the type of problems arising in large-scale finite element calculations in electrocardiology.

METHODS

Region Representation
Typically the geometry information for defibrillation modeling is collected from MRI or CT images. The extracted surfaces are then triangulated. The whole volume is tessellated in three dimensional simplices (tetrahedra). To assure generation of well shaped elements with increased density at the regions of interest, the Delaunay geometric construction is applied to a closed volume [8, 2]. Each region is defined by its exterior surface triangulation and an assigned depth level according to the level tree (the outermost region has the lowest depth level).

Region Classification: Algorithm Outline
To assign the conductivity tensor to each volume element its affiliation to the proper region has to be deter-
mined. The search starts with the region having the lowest depth level using the respective triangulated surface. If an element’s centroid lays to the interior of the boundary surface, the volume element (tetrahedron) is also considered to be interior to that region. The search continues for all domains using only the volume elements in their immediate encompassing region. The inside/outside decision is made using the modified ray-tracing procedure. The general algorithm for region classification is outlined below.

Function: Classify

Inputs:
N_{reg} - number of regions;
N_{tet} - number of tetrahedra (after tessellation);
N_{enc} - number of encounters of the ray with the surface of the current region;
{TET} - list of tetrahedra;
{TET} → Flag - flag, indicating region of affiliation.

begin
  determine RP;
  for current-region ← 1 to N_{reg} do
    find the immediate encompassing region: R_{up};
    for Tetrahedron ← 1 to N_{tet} do
      if (TET→Flag ≡ R_{up}) then
        Ray”Tube”Trace;
        if ((N_{enc} mod 2) ≠ 0) then
          TET ∈ current region;
      end
  end
end

Abbreviations:
RP - reference point,
  can be any point outside the outermost surface;
CT - centroid of the current tetrahedron;
Ray - the line \{RP → CT\};

Ray”Tube”Tracing: Algorithm Outline

In the traditional ray tracing, the ray is formed by a reference point at “infinity”, or simply outside the outermost region, and the centroid of any of the tetrahedra. Its intersection point with the plane of any of the surface triangles is tested and counted only if it is inside the respective triangle. If the ray hits a vertex or crosses an edge it is perturbed in order to prevent miscounting. Finally, an even number of counts indicates a point outside, an odd number of counts indicates a point inside (figure 1). As shown below, computationally expensive operations have to be performed for each surface triangle to classify each volume element in each region.

We modified the ray tracing procedure confining the search to the vicinity of the ray by considering the following:
(a) the centroid of a surface triangle intersected by the ray lays in a cylindrical region around the ray with a radius equal to \( \frac{\sqrt{3}}{3} \) of its longest median;
(b) the longest median in the triangle is always less than its longest side.

Considering (a) and (b) we can derive a criterion for excluding surface triangles from the search for intersection. The squared radius of the circular region that remains to be considered is \( \text{coeff} \times \text{squared max side length} \) (figure 2), where \( \text{coeff} = (\frac{\sqrt{3}}{3})^2 \approx 0.45 \).

The test operations are by far less computationally expensive than the actual intersection search, thus speeding the algorithm.

Function: Ray”Tube”Trace

Inputs:
N_{tri} - number of surface triangles for the current region;

begin
  for Triangle ← 1 to N_{tri} do
    determine \( d^2 \{CTri, Ray\} \);
    find the Tube;
    if (Triangle ∈ Tube) then
      construct the Plane;
      if (Ray ∩ Plane ≠ ∅, i.e. IP∃) then
        if (IP ∈ Line.Segment \{RP, CT\}) then
          if (IP ∈ Triangle) then
            increment N_{enc};
          end
        end
      end
    end
  return N_{enc};
end

Abbreviations:
CTri - centroid of the current surface triangle;
Plane - plane of a surface triangle,
in normal form: \( ax + by + cz + d = 0 \);
IP - intersection point \{Ray ∩ Plane\};
\( d^2 \{CTri, Ray\} \) - point-line squared distance;
Figure 2: Ray "tube" tracing. Surface triangles inside and outside the tube are shown.

*Tube* - cylindrical region around the Ray, where the intersection is probable.

**Application to Defibrillation Modeling**

The classification technique was incorporated into a software for automatic generation of unstructured three dimensional finite element grids, specifically developed for defibrillation modeling.

The data for the volume conductor model were collected from 90 transverse MRI images of a human torso. The extracted boundary information was used to triangulate the surfaces and then the volume was tessellated and regions were classified as described above. Finite element analysis was performed on models of transvenous defibrillation.

**RESULTS**

The whole thorax with the internal structures used in the defibrillation models is shown on figure 3.

Table 1 contains information by region for a mesh of 14 complex regions in the thorax. The rows of the table list the: regions, the number of boundary triangles, the percentage of triangles in the "tube" and the speedup (in times, compared to the traditional ray trace). The numbers in the last two rows are normalized for the number of tetrahedra (the gain is linear in mesh size). The percentage of triangles in the tube varied by region and the average value was found to be $(7.23\pm3.67)\% (n=14)$. The speedup for the surfaces examined was: $(10.01\pm3.45)$ times$(n=14)$.

We tested the sensitivity of the tube area to the position of the reference point. By moving the reference point to each of the vertices of the cubic convex hull, enclosing the region we obtained the following percentage triangles in the tube for the epicardium: $(5.23\pm1.1)\% (n=8)$.

Figure 3: Regions in a realistic geometry of the thorax

<table>
<thead>
<tr>
<th>potential [V]</th>
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<tbody>
<tr>
<td>5.0</td>
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<tr>
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Figure 4: Potential field in the heart in transvenous defibrillation (wire electrodes in the right ventricle and in superior vena cava)
Table 1: Results for a mesh of the thorax with 14 regions. SVC=superior vena cava; LV/RV=left/right ventricle; LA/RA=left/right atrium; LL/RL=left/right lung; PA=pulmonary artery; skm=skeletal muscle; e-de=electrode; n=boundary triangles. The speedup is given in times, compared to traditional ray trace.

<table>
<thead>
<tr>
<th>region</th>
<th>torso</th>
<th>skm</th>
<th>LL</th>
<th>RL</th>
<th>SVC</th>
<th>SVC e-de</th>
<th>epi</th>
<th>LV</th>
<th>RV</th>
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<th>RA</th>
<th>LA</th>
<th>aorta</th>
<th>PA</th>
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<td>( \Delta )</td>
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<td>960</td>
<td>420</td>
<td>420</td>
<td>300</td>
<td>78</td>
<td>616</td>
<td>168</td>
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<td>78</td>
<td>144</td>
<td>120</td>
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<tr>
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<td>3.1</td>
<td>6.7</td>
<td>5.9</td>
<td>5.3</td>
<td>5.6</td>
<td>4.9</td>
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<td>9.6</td>
<td>12.9</td>
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</tr>
<tr>
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<td>12.4</td>
<td>9.3</td>
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<td>5</td>
<td>5</td>
<td>10</td>
<td>8</td>
<td>15</td>
<td>13.7</td>
<td>6</td>
</tr>
</tbody>
</table>

The calculated potential field on the constructed mesh resulting from a transvenous defibrillation is shown on figure 4. Realistic heart, blood vessels and electrode geometry are included.

DISCUSSION

Algorithm Cost Analysis

Two major costs are involved in our search algorithm. (a) cost of performing ray-object intersection tests, which are confined to the "tube" \( C_{\text{is}}(\text{tube}) \); (b) cost of closeness test, i.e. "tube" construction \( C_{\text{test}} \).

The significant reduction in the first (compared to the cost for the whole boundary, \( C_{\text{is}}(\text{all}) \)) is partially masked by the presence of the second.

We estimated the above costs empirically. The ratio \( \frac{C_{\text{is}}(\text{all})}{(C_{\text{is}}(\text{tube}) + C_{\text{test}})} \) is directly related to the obtained speedup and is shown in table 1.

From the results we conclude that the area of the "tube" and the resulting speedup depend on the particular surface triangulation and shape and slightly on the position of the reference point, but for all 14 surfaces examined: \( C_{\text{is}}(\text{all}) \gg (C_{\text{is}}(\text{tube}) + C_{\text{test}}) \). The resulting speedup is significant and comparable to the results obtained by hierarchical bounding boxes [7].

Application to Defibrillation Modelling

The outlined region classification algorithm in conjunction with a volume tetillation program can be used as a fast tool for realistic modeling of complex heterogeneous domains. Precise potential mapping, resulting from defibrillation induced fields can be obtained and used to optimize electrode size and position.

CONCLUSIONS

A modified ray tracing procedure for region classification in highly heterogeneous objects is developed and implemented. It is suitable for the purposes of large-scale finite element modeling in biological systems or other applications requiring efficient region classification. The advantages of the proposed method over existing improved ray tracing approaches can be summarized as follows: (1) it is simpler and straightforward to implement; (2) it still gives a significant speedup, that is linear in mesh size; (3) it is independent of the object structure and shape (specifically appropriate for non-convex regions, such as the lungs, the great blood vessels, etc.). The region classification is often the most time consuming part of the automatic finite element grid generation, thus the proposed technique can be a valuable asset in all modeling studies on complex domains.

REFERENCES


